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Abstract. The gravity field dedicated satellite missions like CHAMP, GRACE, and GOCE are supposed to map the Earth's global gravity field with unprecedented accuracy and resolution. New models of Earth's static and time-variable gravity field will be available every month as one of the science products from GRACE. Here we present an alternative method to estimate the gravity field efficiently using the in situ satellite-to-satellite observations at the altitude and show results on static as well as temporal gravity field recovery. Considering the energy relation between the kinetic energy of the satellite and the gravitational potential, the disturbing potential difference observations can be computed from the orbital parameter vectors in the inertial frame, using the high-low GPS-LEO GPS tracking data, the low-low satellite-to-satellite GRACE measurements, and data from 3-axis accelerometers (Jekeli, 1999). The disturbing potential observation also includes other potentials due to tides, atmosphere, other modeled signals (e.g. N-body) and the geophysical fluid signals (hydrological and oceanic mass variations), which should be recoverable from GRACE mission with a monthly resolution. The simulation results confirm that monthly geoid accuracy is expected to be a few cm with the 160 km resolution (up to degree and order 120) once other corrections are made accurately. The time-variable geoids (ocean and ground water mass) might be recovered with a noise-to-signal ratio of 0.1 with the resolution of 800 km every month assuming no temporal aliasing.

Key words. GRACE mission, Energy integral, Geopotential, Satellite-to-satellite tracking, Temporal gravity field

1 Introduction

By the middle of this decade, measurements from GRACE (Tapley et al., 1996), CHAMP (Reigber et al., 1996) and GOCE (Rummel et al., 1999) gravity mapping missions are expected to provide significant improvement in our knowledge of the Earth's mean gravity field and its temporal component. It is expected that the mean geoid would be improved to one cm accuracy at a wavelength of 100 km or longer (primarily by GOCE), and the time-varying mass variations of the Earth system in terms of climate-sensitive signals could be measured with sub-centimeter accuracy in units of column of water movement near Earth surface with a spatial resolution of 250 km or longer, and a temporal resolution of weeks (primarily by GRACE).

Gravity Recovery and Climate Experiment (GRACE) launched on 17 March 2002 for a mission span of 5 years or longer. The mission consists of two identical co-orbiting spacecrafts with a separation of 220 ± 50 km at a mean initial orbital altitude of 500 km with a circular orbit and an inclination of 89° for near-global coverage. The scientific objectives of GRACE include the mapping and understanding of climate-change signals associated with mass-variations within the solid Earth – atmosphere – ocean – cryosphere – hydrosphere system with unprecedented accuracy and resolution in the form of time-varying gravity field (e.g. Wahr et al., 1998). New models of Earth's static as well as time-variable gravity field will be available every 30 days for a time-span of 5 years.

The dual-one way K- (24.5 GHz) and Ka- (32.7 GHz) band microwave inter-satellite ranging system with a precision of 0.1 μm/sec in range-rate (Kim et al., 2001), the Ultra-Stable Oscillator (USO) accurate to within 70 picosecs of time-tagging, the 3-axis super-STAR accelerometers with a precision of 4×10^{-12} m/s^2 (Davis et al., 1999; Perret et al., 2001) and the dual-frequency 24-channel Blackjack GPS receivers comprise the instrument suite for GRACE's mapping of the global gravity field with unprecedented accuracy and resolution. By detecting the differenced measurements like range rates, the high resolution or small wavelength parts of the gravity field will be amplified, and they have a chance to be recovered from the satellite-borne instruments. Traditionally, the orbital perturbation techniques have been developed and employed to simultaneously solve for the geopotential coefficients as well as other orbital parameters.

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In this study, we use a more straightforward method to estimate the Earth's gravitational harmonic coefficients based on the boundary value problem in the potential theory. The potential difference values between two satellites along the orbit can be computed by combining of the inter-satellite range-rate, position, velocity, and acceleration data through the energy conservation principle (Jekeli, 1999). They are treated as the classical observational boundary values on the fixed boundary, i.e. the orbit. The use of energy conservation principle has been successfully demonstrated by analyzing real CHAMP data in very recent studies (Han et al., 2002; Gerlach et al., 2002; Sneeuw et al., 2002; Visser et al., 2002). Here, we extend to use this approach for GRACE monthly static as well as temporal gravity recovery complete up to degree and order 120 in a very efficient manner. The inversion is based on the conjugate gradient iterative approach and has been demonstrated to be able to efficiently recover the gravity field solutions up to degree and order 120 or more. An appropriate pre-conditioner like a block-diagonal normal matrix (which contains most of the power of the full normal matrix) is used to accelerate the convergence rate. This efficient inversion was first proposed and successfully demonstrated for GOCE satellite gradiometry (Schuh, 1996; Schuh et al., 1996; Ditmar and Klees, 2002). The synthetic potential difference observations were generated with the expected error of GRACE range-rate measurements, and the monthly gravity field was recovered. Assuming no temporal aliasing, two temporal gravity signals including the ocean and ground water mass redistributions were recovered in the presence of the measurement error only.

2 In situ potential difference observable and the efficient inversion

The in situ observable of interest is the potential difference between two satellites expected only from the satellite-to-satellite tracking mission in low-low mode like GRACE. It can be computed by measuring the range-rates, velocity vectors, and position vectors in the inertial frame. The following shows the approximate model, which has been developed and used by Wolff (1969), Rummel (1980), and Jekeli and Rapp (1980):

$$V_{12} = V_2 - V_1 \approx |\dot{\boldsymbol{x}}_1^i| \dot{\rho}_{12} \,, \tag{1}$$

where V_1 and V_2 are the gravitational potentials at the first and second satellite. The term, $|\dot{\boldsymbol{x}}_1^i|$, is the speed of the satellite and $\dot{\rho}_{12}$ is the range-rate between the two satellites. This model relates the in situ inter-satellite range-rate measurements to the gravitational potential difference between two satellites, V_{12}. This model, however, is not appropriate to take full advantage of the current instrument's capability. Especially, it does not include the time-variable effect of gravitational potential due to the Earth rotation, which is significant in the order of $\pm 1\,\mathrm{m^2/s^2}$. Considering some of these significant effects, the new rigorous model was developed by Jekeli via the energy conservation principle (Jekeli, 1999). By correcting the mistakes in Eq. (29) of Jekeli (1999), reformulating it in terms of the disturbing potential (Earth's gravitational potential minus normal gravitational potential) difference, T_{12}, and assuming the energy dissipation term is corrected by measuring non-gravitational accelerations accurately, the correct model is given by:

$$T_{12} = |\dot{\boldsymbol{x}}_1^0| \delta\dot{\rho}_{12} + v_1 + v_2 + v_3 + v_4 + \delta V R_{12} - \delta E_0 \,, \tag{2}$$

where $v_1 = (\dot{\boldsymbol{x}}_2^0 - |\dot{\boldsymbol{x}}_1^0|\boldsymbol{e}_{12}) \cdot \boldsymbol{\delta}\dot{\boldsymbol{x}}_{12}$, $v_2 = (\boldsymbol{\delta}\dot{\boldsymbol{x}}_1 - |\dot{\boldsymbol{x}}_1^0|\boldsymbol{\delta e}_{12}) \cdot \boldsymbol{x}_{12}^0$, $v_3 = \boldsymbol{\delta}\dot{\boldsymbol{x}}_1 \cdot \boldsymbol{\delta}\dot{\boldsymbol{x}}_{12}$, and $v_4 = \frac{1}{2}|\boldsymbol{\delta}\dot{\boldsymbol{x}}_{12}|^2$. The superscript, 0, denotes a quantity based on the known reference field such as GRS80, and the symbol, δ, indicates a residual quantity defined between the true field and the reference field. The sixth term of the right hand side, $\delta V R_{12}$, is the potential rotation difference between two satellites, which can be computed with a linear combination of positions and velocities of two satellites. The last term, δE_0, is the residual energy constant of the system. Without the correct knowledge of this term, zero degree and order harmonic and especially zonal harmonics would be less accurate. This model indicates that the accurate potential difference between two satellites can be obtained by measuring the inter-satellite range rate as well as the position vectors and velocity vectors, which are available from primarily GPS. Again, the on-board accelerometers are assumed to capture all non-conservative forces acting on the satellites, and the dissipating energy is assumed to be corrected accurately.

The expected range rate accuracy from K-band ranging of GRACE mission is about $0.1\,\mu$m/s (Kim et al., 2001; Kim in UT/CSR, private communication, 2002), and this corresponds to the potential difference accuracy in the level of $10^{-3}\,\mathrm{m^2/s^2}$. In order to take full advantage of this high-precision range rate measurements, the commensurate accuracy of a single satellite's position and velocity should be less than 7 cm and $5\,\mu$m/s, respectively, and that of inter-satellite baseline position and velocity should be less than 0.1 mm and $2\,\mu$m/s, respectively, (Jekeli, 1999). These high precision orbital parameters might be obtainable with the aid of high precision range and rage-rate measurements together with the Blackjack class GPS receiver. The registration or coordinatization of the in situ observables causes error as well, because of the imperfect orbit. It, however, is not very sensitive to GRACE potential "difference" observable, because the orbit error of two satellites would be highly correlated (Jekeli and Garcia, 2000). The orbit, therefore, would be fixed when the relationship between the observable and the unknown geopotential coefficients is established.

For one month of data and 10 s sampling rate, the number of observations is a quarter million and the number of parameters is 14 521 for $N_{max} = 120$. In the direct least-squares solution, the huge computations, like accumulating the normal matrix and evaluating the Legendre functions at every observation point, cannot be avoided. This computation sometimes is too time-consuming, especially very high degree model ($N_{max} > 200$) such as GOCE gravity recovery (Klees et al., 2000; Ditmar and Klees, 2002). For the design matrix, $\mathbf{A} \in R^{n \times m}$ with rank$(\mathbf{A}) = m$ and for the

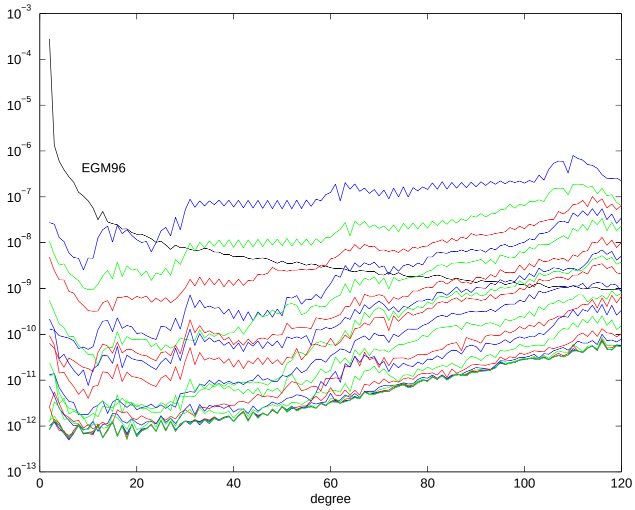

Fig. 1. Square root of averaged degree variances of EGM96 and the errors of intermediate iterates.

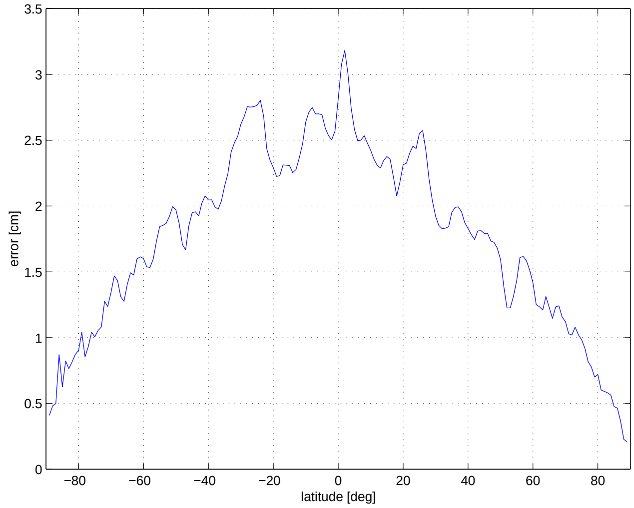

Fig. 2. Monthly mean geoid error from GRACE (RMS over all longitudes.

identity weight matrix, to set up the full normal matrix requires $\sim O(nm^2)$ floating-point operations (flops) and to solve the unknown vector through the Cholesky decomposition requires $\sim O(m^3)$ additional flops. For example, the method via the direct least-squares to estimate the monthly gravity field up to $N_{max} = 120$ requires $O(nm^2 + m^3) \sim 259\,200 \times 121^4 + 121^6 \sim O(10^{14})$ flops. For a serial processing based on CRAY SV1 (500 MHz vector processor) platform, it takes almost 5 days (which is an estimated value) for one month of observations and a 10 s sampling rate for $N_{max} = 120$. In addition, about 800 MB of memory and hard disk space are required to store the upper triangular part of the normal matrix. This CPU wall-clock can be reduced by parallel processing, however it still needs lots of computer resources like huge memory and CPU clocks in the direct least-squares approach. Here we use an alternative approach, i.e. conjugate gradient, which requires only a couple of tens MB of memory and $\sim O(2knm + km^2)$ flops, where k is the number of iterations. For example, k is about 15 for GRACE potential difference observations for $N_{max} = 120$ without any *a priori* constraints like Kaula's rule. Compared with the direct method, the efficiency of the iterative method is dramatically improved by a factor a thousand times less flops. In addition, a twin satellite mission, GRACE, provides a similar block-diagonally dominant normal matrix as a single satellite mission, such as GOCE. The block-diagonal matrix can be used as a pre-conditioner to accelerate the convergence rate of conjugate gradient inversion.

3 Results

The previously developed method is used to recover the global gravity field using the in situ potential difference observables expected from GRACE mission for one month. The time-variable gravity fields due to the mass redistribution of the ocean and ground water are studied, and their long wavelength (resolution $\geq$ 800 km) parts were successfully recovered from GRACE in the presence of measurement error assuming no temporal aliasing.

3.1 Static gravity field recovery

For GRACE simulation, the two satellites' perturbed orbits were generated using the reference gravity field EGM96. The initial altitude was about 400 km and the initial satellite separation was around 200 km. The inclination was 89 degrees, and the near circular orbit was assumed. Along these perturbed orbits, the simulated observations were computed using EGM96 gravity model truncated at degree and order 120. Then, they were corrupted by the random noise with the standard deviation, $\sigma = 10^{-3}\,\mathrm{m}^2/\mathrm{s}^2$ (the corresponding range-rate error is about 0.1 μm/s). The 30 days of observations were regularly sampled every 10 s. The observations based on these realistic orbits do not provide a block-diagonal normal matrix, but a fully occupied normal matrix. If one accumulates and inverts this huge normal matrix, the problem is straightforward. However, it could be very time-consuming with the true normal matrix. Therefore, the iterative method was used to estimate the global geopotential coefficients up to degree and order 120 every month.

The conjugate gradient produced the intermediate iterates and there was no significant improvement in the estimates after 15 iterations starting with zeros as initial values. No *a priori* constraint like Kaula's rule was used. Each iteration takes about 20 min in terms of CPU wall-clock time. The entire procedure would take less than 8 h in CPU wall-clock time to prepare a preconditioner, process one month of data, and determine the geopotential coefficients up to $N_{max} = 120$. In order to assess the intermediate coefficients every i-th iteration step, we used the square root of averaged error degree variances depicted in Fig. 1. This value indicates the

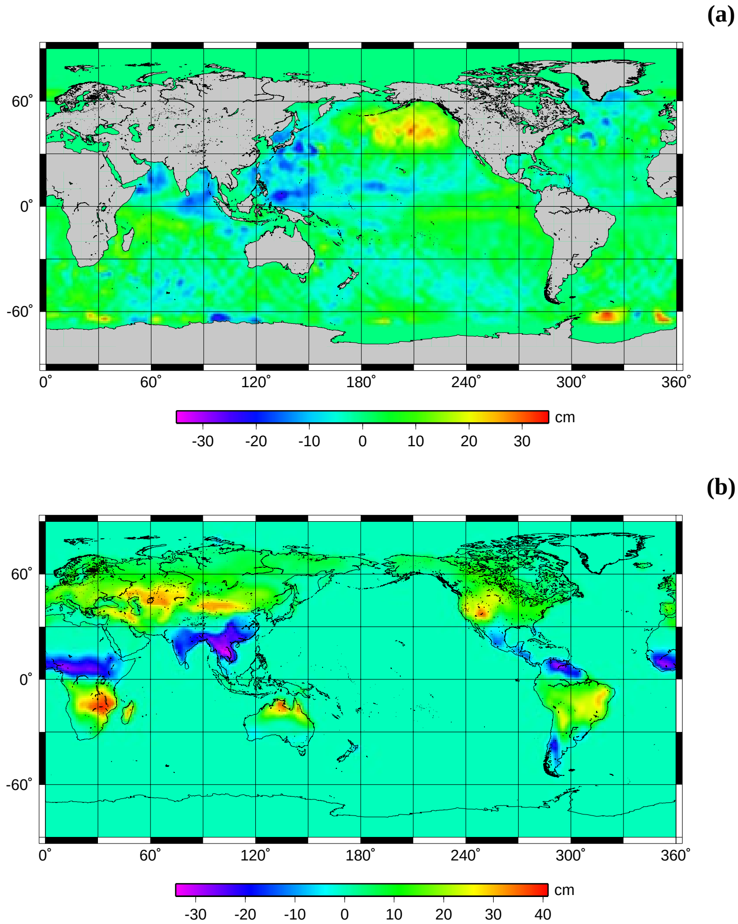

Fig. 3. **(a)** Monthly averaged ocean mass redistribution during T/P Cy. 196-198; **(b)** Monthly averaged water storage anomaly on February 1993.

average magnitude of the error per degree of the i-th iterate. For comparing the magnitude of the error and the signal, the degree RMS of EGM96 signal was computed and depicted in the same figure. The estimates after the 1st iteration have an error, whose magnitude is larger than the magnitude of the signal after degree 30. However, the solutions gradually converge to the true solution. After 15 iterations, there was no significant improvement.

Using the final iteration, the geoid height was computed. The 'truth' geoid was calculated using EGM96 truncated at degree and order 120. Figure 2 shows the RMS of the geoid height error over all longitudes, which is a function of latitude. The error decreases away from the equator because the satellite orbit is converged and the number of observations increases toward the poles. 1 to 3 cm geoid with the 160 km resolution (up to degree and order 120) seems to be possible every month from the GRACE mission.

3.2 Time-variable gravity field recovery

The mass changes between the Earth's surface and the satellite altitude, like ocean, atmosphere and ground water hydrology, generate the anomalous gravity signals with respect to a certain mean (static) gravity field. GRACE is projected to recover the long wavelength part of those mass changes by detecting their gravitational effect, i.e. range, range-rate, and consequently potential difference between two satellites. For this simulation, two different sources of surface mass changes, i.e. ocean and ground water, were considered, and their gravitational effects were computed in terms of spherical harmonic coefficients. We used the quadrature equation same as Wahr et al. (1998) and Hwang (2001) to compute the spherical harmonic coefficients based on the regular gridded data in terms of the height of the water column (called the equivalent water thickness).

$$\begin{Bmatrix} \Delta C_{nm} \\ \Delta S_{nm} \end{Bmatrix} = \frac{3(1+k_n)\sigma_w}{4\pi R\sigma_E(2n+1)} \iint \Delta h(\theta,\lambda)\overline{P}_{nm}(\cos\theta) \begin{Bmatrix} \cos m\lambda \\ \sin m\lambda \end{Bmatrix} \sin\theta d\theta d\lambda \,, \quad (3)$$

where ΔC_{nm} and ΔS_{nm} are the spherical harmonic coefficients of time-variable surface mass change, which will be estimated by the GRACE mission. k_n is the load Love number of degree n that describes the Earth's elasticity, σ_w is the

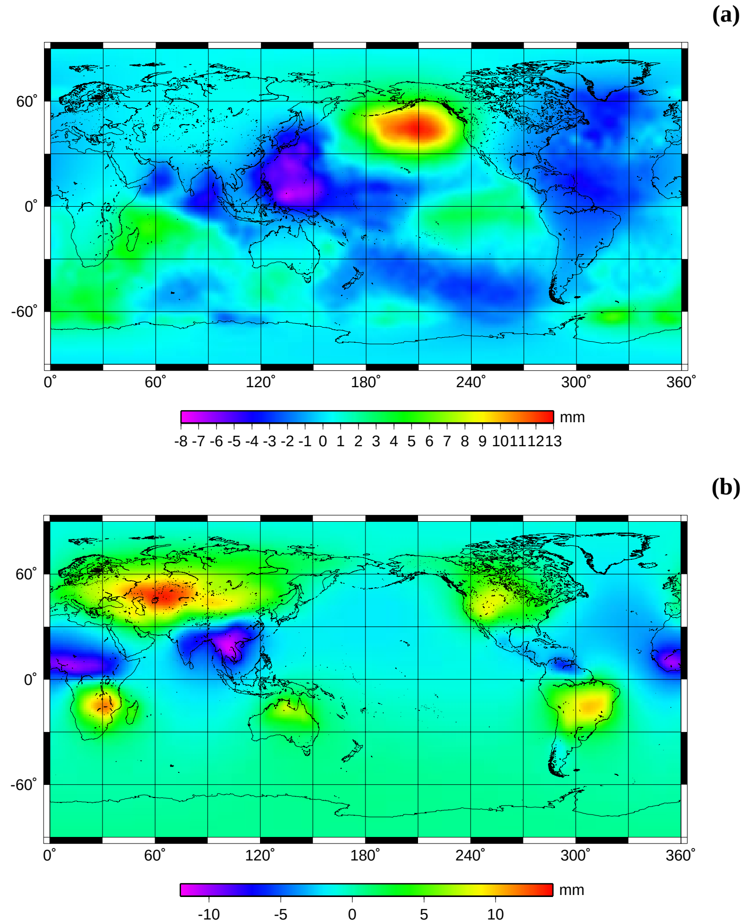

Fig. 4. **(a)** Geoid change due to monthly averaged ocean mass redistribution during T/P Cy. 196-198; **(b)** Geoid change due to monthly averaged ground mass redistribution on February 1993.

density of water, σ_E is the average density of the Earth, and $\Delta h(\theta, \lambda)$ is the equivalent water thickness. The equivalent water thickness is the expression of anomalous surface mass in terms of water height and it is computed by dividing the surface density (mass per area) of anomalous mass by the volume density (mass per volume) of the water. Then, the geoid height due to the surface mass change can be computed using the determined coefficients.

In order to compute the ocean mass change over one month, the corrected sea level anomaly (CSLA) was calculated by subtracting mean sea surface height (MSS) and the steric effect (thermal expansion and salinity change of the ocean) from sea surface height (SSH) for that month. The CSLA is the real ocean mass movement at a particular period with respect to MSS (Hwang, 2001). The MSS was computed by averaging 6 years T/P altimeter data and the one month SSH was computed using T/P data from Cyc. 196 through 198 (January, 1998). The steric effect was computed using the temperature data of each ocean layer at corresponding period.

A monthly mean continental water storage field was computed from the two layers (0–10, 10–200 cm) CDAS-1 soil moisture data and snow accumulation data from January 1993 to December 1998. Global data, except the polar region, are available in the form of the equivalent water thickness from the web site in the University of Texas (GGFC, 2002). The monthly water storage anomaly (WSA) was computed by subtracting 6 years mean water storage (MWS) from water storage (WS) on February 1993. The WSA represents the ground water mass movement at a particular period with respect to MWS.

Figures 3a and b show the mean anomalous ocean and ground water mass redistribution for a certain month. Their magnitudes are on the decimeter level. Based on these data, the spherical harmonic coefficients up to degree and order 60 were computed applying Eq. (3). The Earth's static geoid is distorted due to this anomalous mass redistribution on the Earth surface. The overall effect is on the level of a couple of millimeters, which were depicted in Figs. 4a and b.

In the spectral domain the time-variable geoid was analyzed. Figure 5 shows the amplitudes of the error in the GRACE monthly gravity estimates and the signal of CSLA

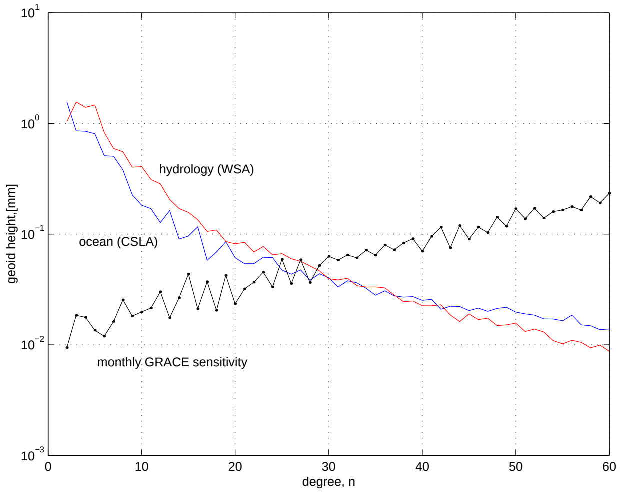

Fig. 5. Geoid degree variances of CSLA and WSA and GRACE monthly sensitivity.

and WSA versus the degree in terms of the geoid height. The amplitude of both temporal geoid spectra tend to decrease as the degree increases, while the amplitude of GRACE estimates error spectrum tends to increase as the degree increases. Finally, the temporal gravity signal and one month GRACE error intersect around the degree 26. It indicates that the low degree part (degree and order ≤ 25, of which resolution is about 800 km) of CSLA and WSA can be recovered using monthly GRACE data by separating the monthly time-variable part from recovered monthly gravity field assuming we have accurate reference static gravity field like 5 or 6 years mean gravity field.

The temporal geoid or mass variations affect the GRACE in situ observable like the range, range rate, and potential difference between two satellites, and the low degree part of them are recoverable on the monthly data basis. Generated were the one month synthetic observations using EGM96 ($N_{max} = 120$) combined with time-variable gravity field ($N_{max} = 60$). Two sources of time-variable field, CSLA and WSA, were tested separately. Finally, the observable was corrupted with the same random noise as before. Then, the spherical harmonic geopotential coefficients for that month were estimated. The estimates include both static and time-variable gravity fields, therefore the static part, i.e. EGM96, was removed to obtain the temporal gravity part only.

Figures 6a and b show that the recovered geoid heights due to the ocean (CSLA) and ground hydrology (WSA). The standard deviations of the geoid signal due to CSLA and WSA for one month were 2.0 mm and 2.8 mm, respectively. The standard deviations of the geoid difference between the true and recovered one were 0.2 mm and 0.2 mm. The time-variable geoids were recovered within the noise-to-signal ratio of $0.1 \sim 0.07$ with the resolution of 800 km every month. However, caution should be made on the fact that the input temporal gravity signals were monthly mean fields, so there was no aliasing coming from ocean and hydrology in this simulation. In the real case, the problem would be more complicated because ocean and hydrology are not static phenomena in a month. For more time-variable gravity field studies, we refer Wahr et al. (1998), Pail et al. (2000), and Peters (2001).

4 Conclusion and discussion

We have discussed the use of the GRACE in situ potential difference observable to recover the global gravity field accurately. This observable is based on the energy conservation principle, and the accurate orbital parameters are necessary to take a full advantage of high precision range-rate measurements. To apply this principle and to use the potential observables for the global gravity field recovery has been successfully demonstrated in very recent studies using real CHAMP satellite data. The recovered geoid using monthly CHAMP data has a comparable accuracy as other previous gravity models. This approach has many advantages, because it is a more direct approach requiring no integration of the equations of motion, all observables are used as in situ measurements, and it allows alternate correction models, e.g. tides or atmosphere, to be efficiently used to assess their accuracies (e.g. modeling errors or aliasing effects), and to validate the GRACE data product. For the purpose of fast monthly mean gravity recovery, the conjugate gradient method was used to invert one month of GRACE data efficiently. It avoids the massive computation of the normal matrix and gives the solution iteratively and very efficiently. In addition, it allows to use data at the exact measurements points without any data manipulation like interpolation or reduction, which were sometimes used to make the normal matrix more tractable and easily computable in a space-wise approach.

Based on the monthly GRACE simulation study, the cumulative geoid was obtained with an accuracy of a few cm and with a resolution of 160 km (the corresponding degree is 120) every month, once other geophysical fluid mass corrections like tide and atmosphere are done correctly. Concerning the study of the temporal gravity field recovery, the ocean mass and ground water mass redistributions were computed using T/P altimeter measurements and hydrology data from the University of Texas, respectively. The resulting recovered geoid errors were about 0.2 mm with a resolution of 800 km (the corresponding degree is 25) in both cases, while the standard deviations of ocean and hydrological mass transport signals were 2.0 mm and 2.8 mm, respectively. However, it should be mentioned that other effects like ocean tidal and atmospheric modeling error and aliasing should be studied, in order to quantify how large they are, for the successful recovery of temporal gravity signals.

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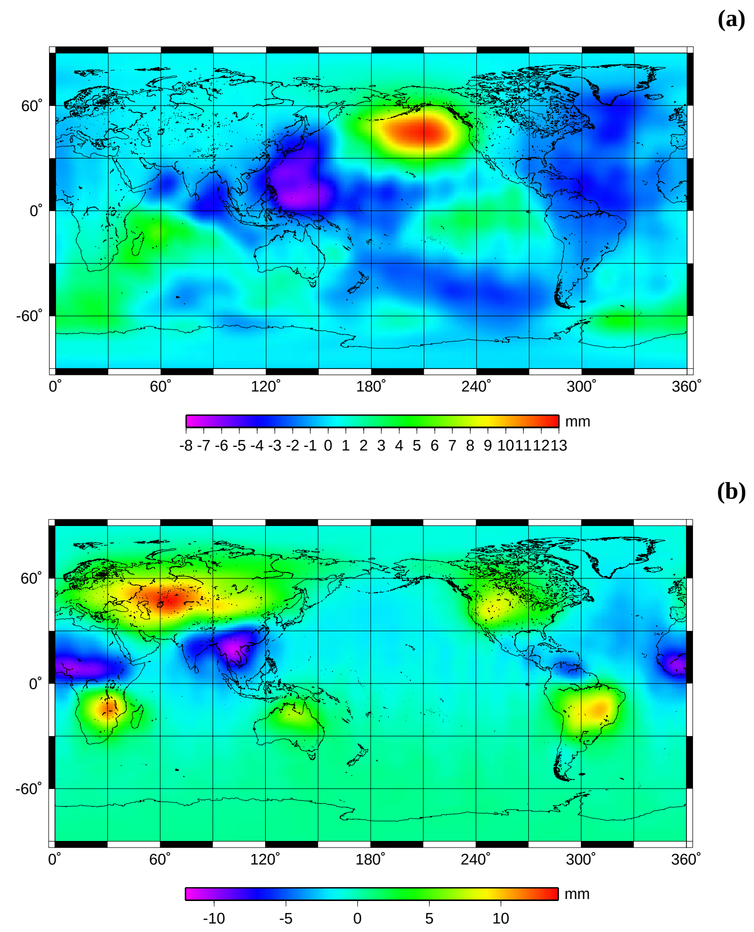

Fig. 6. (a) Recovered geoid due to CSLA (Nmax=25) from GRACE one month data; **(b)** Recovered geoid due to WSA ($N_{max} = 25$) from GRACE one month data.

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